State Estimation for Nonlinear Systems using Restricted Genetic Optimization

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Abstract. In this paper we describe a new nonlinear estimator for filtering systems with nonlinear process and observation models, based on the optimization with RGO (Restricted Genetic Optimization). Simulation results are used to compare the performance of this method with EKF (Extended Kalman Filter), IEKF (Iterated Extended Kalman Filter), SNF (Second-order Nonlinear Filter), SIF (Single-stage Iterated Filter) and MSF (Monte-Carlo Simulation Filter) in the presence of diferents levels of noise.

1 Introduction

Whenever the state of a system have to be estimated from noisy sensor information, some kind of state estimator is employed to fuse together the data from different sensors to produce an accurate estimate of the true system state. If the system dynamics and the observation model are linear, then the minimum mean square estimate can be computed using the *Kalman Filter*. In the non-linear case the filtering problem in the state space is given by

$$x(k) = g(k, x(k-1), \varepsilon(k)), \tag{1}$$

$$z(k) = h(k, x(k), \eta(k)), \tag{2}$$

where $\varepsilon(k)$ and $\eta(k)$ are the disturbances, that are supposed to be independently distributed random vectors:

$$\begin{pmatrix} \varepsilon(k) \\ \eta(k) \end{pmatrix} \sim \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} Q(k) & 0 \\ 0 & R(k) \end{pmatrix} \right) \ . \tag{3}$$

The optimal estimation implies the description of the conditional probability density

$$\hat{x}^{MMSE}(k|k) = E(x(k)|Z^k) = \int x(k)\, p(x(k)|Z^k) dx \ . \tag{4}$$

Unfortunately, this description needs an infinite number of parameters. For this reason, a number of suboptimal approachs have been proposed that use analitic aproximations of the probability distributions, the transition states equation or the measure equation. There are other methods, such as Monte-Carlo method, which need thousands of points in order to aproximate the conditional probability density. In aplications with many dimensions, these methods are not practical. For this reason methods with a reasonable number of operations are needed, such as the filters estudied in this work.

2 The Filtering Problem

The problem is to find an estimation of the state x(k) of the system of interest, with a non-linear discrete dynamics given by

$$x(k) = g(k, x(k-1), \varepsilon(k)) \ . \tag{5}$$

where g is the state function model, x(k) is the state of the system at time k and $\varepsilon(k)$ is the process noise.

The only information available of the system is the noisy observation given by the nonlinear measure equation

$$z(k) = h(k, x(k), \eta(k)) \ . \tag{6}$$

where $z(k)$ is the observation vector, h is the observation model that transforms the state space into the observation space and $\eta(k)$ is the measurement noise. The MMSE estimate coincides with the conditional mean. Let $\hat{x}(i|j) = E[x(i)|Z^j]$ with $Z^j = \{z(1), z(2), ..., z(j)\}^T$. The estimated covariance is $P_{xx}(i|j) = E[\{x(i) - \hat{x}(i|j)\}\{x(i) - \hat{x}(i|j)\}^T|Z^j]$. These equations are difficult to evaluate in practice. For this reason the recursive estimators are employed . In the linear case with additive gaussian noise the MMSE is the Kalman Filter. The linear updated equations in this case are

$$\hat{x}(k|k) = \hat{x}(k|k-1) + W(k)\tilde{z}(k), \tag{7}$$

$$P_{xx}(k|k) = P_{xx}(k|k-1) - W(k)P_{\nu\nu}(k|k-1)W^T(k) \ . \tag{8}$$

The vector $\tilde{z}(k)$ is called the innovation, which is the difference between the observation and the prediction:

$$\tilde{z}(k) = z(k) - \hat{z}(k|k-1) \ . \tag{9}$$

The covariance of this quantity is

$$P_{\nu\nu}(k|k-1) = P_{zz}(k|k-1) + R(k), \tag{10}$$

and $W(k)$ is the Kalman gain and its value is

$$W(k) = P_{xz}(k|k-1)P_{zz}^{-1}(k|k-1) \ . \tag{11}$$

3 Traditional Nonlinear Filters

3.1 EKF and SNF

The Extended Kalman filter is similar to a linearized Kalman Filter , with the exception that the linearization is performed on the estimated trajectory instead of a previously calculated nominal trajectory. For that reason, the functions $g(k, x(k-1), \varepsilon(k))$ and $h(k, x(k), \eta(k))$ are expanded in Taylor series around $\hat{x}(k|k)$ with terms up to first or second order to yield the EKF or SNF respectively. The expansion with second order terms of the transition equation is:

$$
\begin{aligned}
&x(k) = g(k, x(k-1), \varepsilon(k)) \simeq g(k, \hat{x}(k|k-1), 0) \\
&+g_x(k, x(k-1), \varepsilon(k))(x(k-1) - \hat{x}(k-1|k-1)) + g_\varepsilon(k, x(k-1), \varepsilon(k))\varepsilon(k) \\
&+\frac{1}{2}\sum_{j=1}^{n} e_j (x(k-1) - \hat{x}(k-1|k-1))' g^j_{xx}(k, x(k-1), \varepsilon(k))(x(k-1) - \hat{x}(k-1|k-1)) \\
&+\frac{1}{2}\sum_{j=1}^{n} e_j \varepsilon'(k) g^j_{\varepsilon\varepsilon}(k, x(k-1), \varepsilon(k))\varepsilon(k) \\
&+\sum_{j=1}^{n} e_j (x(k-1) - \hat{x}(k-1|k-1))' g^j_{x\varepsilon}(k, x(k-1), \varepsilon(k))\varepsilon(k) \quad .
\end{aligned}
\tag{12}
$$

and the expansion with second order terms of the measure equation is:

$$
\begin{aligned}
&z(k) = h(k, x(k), \eta(k) \simeq h(k, \hat{x}(k|k-1), 0) \\
&+h_x(k, x(k-1), \eta(k))(x(k-1) - \hat{x}(k-1|k-1)) + h_\eta(k, x(k-1), \eta(k))\eta(k) \\
&+\frac{1}{2}\sum_{j=1}^{n} e_j (x(k-1) - \hat{x}(k-1|k-1))' h^j_{xx}(k, x(k-1), \eta(k))(x(k-1) - \hat{x}(k-1|k-1)) \\
&+\frac{1}{2}\sum_{j=1}^{n} e_j \eta'(k) h^j_{\eta\eta}(k, x(k-1), \eta(k))\eta(k) \\
&+\sum_{j=1}^{n} e_j (x(k-1) - \hat{x}(k-1|k-1))' h^j_{x\eta}(k, x(k-1), \eta(k))\eta(k) \quad .
\end{aligned}
\tag{13}
$$

where e_j is the j^{th} Cartesian basis vector.

3.2 IEKF

The measurement prediction, up to first order, is $\hat{z}(k|k-1) = h(k, \hat{x}(k|k-1)$. There are errors prediction errors in using $\hat{x}(k|k-1)$ for $x(k)$. Other additional errors are due to measurement nonlinearity. Its possible to alleviate these errors if the updated state is not computed as an approximate conditional mean, but a maximum a posteriori estimate.

The conditional probability density function, PDF, of $x(k)$ given Z^k, if all the random variables are Gaussian, is

$$
\begin{aligned}
p(x(k)|Z^k) &= p(x(k)|z(k), Z^{k-1}) \\
&= \tfrac{1}{c}\, p(z(k)|x(k))\, p(x(k|Z^{k-1})) \\
&= \tfrac{1}{c}\mathcal{N}(z(k); h(k, x(k)), R(k))\, \mathcal{N}(x(k); \hat{x}(k|k-1), P(k|k-1)) \quad .
\end{aligned}
\tag{14}
$$

Maximizig this function is equivalent to minimizing the following

$$
\begin{aligned}
V(x(k)) = &\tfrac{1}{2}(z(k) - h(k, x(k)))' R(k)^{-1}(z(k) - h(k, x(k))) \\
&+\tfrac{1}{2}(x(k) - \hat{x}(k|k-1))'(P(k|k-1)^{-1}(x(k) - \hat{x}(k|k-1)) \quad .
\end{aligned}
\tag{15}
$$

The Iterated Extended Kalman Filter method (IEKF) uses a Newton-Raphson algorithm to estimate $\hat{x}(k|k)$. Expanding V in a Taylor series up to second order about the i-th estimate of x(k) results in:

$$V = V^i + V_x^{i'}(x - x^i) + \frac{1}{2}(x - x^i)' V_{xx}^i (x - x^i) \ . \tag{16}$$

Setting the gradient to zero:

$$x^{i+1} = x^i - (V_{xx}^i)^{-1} V_x^i \ . \tag{17}$$

This yields the IEKF equation:

$$\begin{aligned} \hat{x}^i(k|k) = \hat{x}^{i-1}(k|k) + P^{i-1}(k|k) H^{i-1}(k)' R(k)^{-1} \{z(k) - h(k, \hat{x}^{i-1}(k|k))\} \\ - P^{i-1}(k|k) P(k|k-1)^{-1} (\hat{x}^i(k|k) - \hat{x}(k|k-1)), \end{aligned} \tag{18}$$

with $H^{i-1}(k) = h_x(k, \hat{x}^i(k|k))$.

3.3 SIF

This technique is concerned with maximizing the *a posteriori* probability density. In the derivation of the EKF, the nominal trajectory is linearized about a nominal trajectory determined by $\hat{x}(k-1|k-1)$and a nominal state $\hat{x}(k|k-1)$. If the nominal trajectory and state are not close to the true trajectory, the truncated expansions of the EKF would represent poor approximations.

$$\begin{gathered} \hat{x}^i(k|k-1) = g^i(\hat{x}(k|k) + g_x^i(k, x(k|k-1, \varepsilon(k)))(\hat{x}(k-1|k-1) - \hat{x}(k|k-1), \\ P_{xx}^i(k|k-1) = g_x^i(k, x(k|k, \varepsilon(k)))) P_{xx}^i(k|k-1) g_x^i(k, x(k|k-1, \varepsilon(k))))' + Q(k), \\ \hat{z}^i(k|k-1) = h^i(k, x(k|k-1), \eta(k)), \\ P_{zz}^i(k) = Hu^i(k, \hat{x}(k|k), x) P_{xx}^i(k, k-1)(Hu^i(k, x(k|k), x))' + R, \\ \tilde{z}^i(k) = z(k) - \hat{z}^i(k|k-1), \\ W^i(k) = P_{xx}^i(k, k-1) h_x^i(k, \hat{x}(k|k), x)' P_{zz}^i(k), \\ P_{xx}^i(k, k) = P_{xx}^i(k, k-1) - W^i(k) Pzz^i(k) W^i(k)', \\ x^i(k|k) = x^i(k, k-1) + W^i(k)\tilde{z}(k) \ . \end{gathered} \tag{19}$$

3.4 MSF

Tanizaki and Mariano proposed an algorithm based on the Monte-Carlo stochastic simulations, where the normal random numbers are generated for the error terms $\varepsilon(k)$ and $\eta(k)$ and the state variables $x(k)$ and $x(k-1)$ more precisely. This approach is based on the equations:

$$\begin{aligned} x(k) = g(k, \hat{x}(k|k-1), 0) \\ + g_x(k, x(k-1), \varepsilon(k))(x(k-1) - \hat{x}(k-1|k-1)), \end{aligned} \tag{20}$$

and

$$\begin{aligned} z(k) = h(k, \hat{x}(k|k-1), 0) \\ + h_x(k, x(k-1), \eta(k))(x(k-1) - \hat{x}(k-1|k-1)) \ . \end{aligned} \tag{21}$$

This algorithm is a combination of EKF and Monte-Carlo stochastic simulations. The equations of the Kalman filter are approximated by random draws:

$$\begin{gathered}
\hat{x}(k|k-1) = \frac{1}{n}\sum_{i=1}^{n}\hat{x}_i(k|k-1), \\
P_{xx}(k|k-1) = \frac{1}{n}\sum_{i=1}^{n}(\hat{x}_i(k|k-1) - \hat{x}(k|k-1))(\hat{x}_i(k|k-1) - \hat{x}(k|k-1))', \\
\hat{z}(k|k-1) = \frac{1}{n}\sum_{i=1}^{n}\hat{z}_i(k|k-1), \\
P_{zz}(k|k-1) = \frac{1}{n}\sum_{i=1}^{n}(\hat{z}_i(k|k-1) - \hat{z}(k|k-1))(\hat{z}_i(k|k-1) - \hat{z}(k|k-1))', \\
P_{xz}(k|k-1) = \frac{1}{n}\sum_{i=1}^{n}(\hat{z}_i(k|k-1) - \hat{z}(k|k-1))(\hat{x}_i(k|k-1) - \hat{x}(k|k-1))', \\
W(k) = P_{xz}(k|k-1)'P_{zz}(k|k-1), \\
P_{xx}(k|k) = P_{xx}(k|k-1) - W(k)P_{zz}(k|k-1)W(k)',
\end{gathered} \tag{22}$$

and

$$\hat{x}(k|k) = \hat{x}(k|k-1) + W(k)(\hat{z}_i(k|k-1) - \hat{z}(k|k-1)), \tag{23}$$

where $\hat{x}^i(k|k-1) = g(k, \hat{x}^i(k-1, k-1), \varepsilon^i(k))$, and $\hat{z}^i(k, k-1) = h(k, \hat{x}^i(k, k-1), \eta^i(k))$.

4 The New Filter

The genetic algorithm is a probabilistic process of search based on the natural selection and the genetic laws. The population $\mathcal{J} = (\mathcal{J}_1, \mathcal{J}_2, ..., \mathcal{J}_n) \in J^N$ is modified according to the natural evolutionary process: after initialization, selection $\omega : J^N \longrightarrow J^N$, cross $\chi : J^N \longrightarrow J^N$, and mutation $\Xi : J^N \longrightarrow J^N$ are executed recursively in a loop. Each run of the loop is called a *generation* and $\mathcal{J}$ denotes the population at generation τ.

The selection operator is intended to improve the average quality of the population by giving individuals of higher quality a higher probability to be copied into the next generation. The selection therefore focuses the search on promising regions in the search space. The quality of an individual is measured by a fitness function $f : J \longrightarrow \mathbb{R}$, where J represents the space of all possible individuals.

The genetic algorithms are often used as a global optimization method of time independent functions, and usually executed off line. However, the natural selection is a local or semi local process in which the species adapt themselves to the environment, which is in turn time dependent (it is on line).

It is possible to adapt the method of genetic algorithms if the search is restricted to a neighborhood of the previous estimation using as fitness function: $f : B(\hat{(}k|k-1), \sigma) \longrightarrow \mathbb{R}$, with $\sigma = \|P(k|k-1)\|$,

$$f(J) = \frac{1}{10^{-8} + V(\hat{x}(k|k-1))} \tag{24}$$

and

$$\begin{aligned}
V(x(k)) = {} & \tfrac{1}{2}(z(k) - h(k, x(k)))'R(k)^{-1}(z(k) - h(k, x(k))) \\
& + \tfrac{1}{2}(x(k) - \hat{x}(k|k-1))'(P(k|k-1)^{-1}(x(k) - \hat{x}(k|k-1)) \ .
\end{aligned} \tag{25}$$

Fig. 1. Restricted Genetic Optimization flowchart1.

Therefore, the algorithm updates the estimations of the states and the covariance matrix and the loop is repeated again, as illustrated in figure 1. The conditional PDF of $x(k)$ given Z^k is given as

$$\begin{array}{l} p(x(k)|Z^k) = p(x(k)|z(k),\, Z^{k-1}) = \\ \frac{1}{c}\, p(z(k)|x(k))\, p(x(k|Z^{k-1})) = \\ \frac{1}{c} N(z(k);\, h(k,\, x(k)),\, R(k)) N(x(k);\, \hat{x}(k|k-1),\, P(k|k-1)) \ . \end{array} \tag{26}$$

Maximizing the above function is equivalent to calculating a maximum a posteriori (MAP) estimate. This is also equivalent to minimizing $V(x(k))$, i.e. maximizing the fitness function $f(J)$.

The standard fitness function (i.e. divided by the sum of fitness) is an approximation of the conditional density function

$$p(x(k)|Z^k) = \frac{p(z(k)|x(k))\, p(x(k)|Z^{k-1})}{\int p(z(k)|x(k))\, p(x(k)|Z^{k-1})} \ . \tag{27}$$

From the above it is clear that it is possible to calculate accurately the non linearities of the functions f and g, however introducing the hypothesis of Gaussian noise can not be avoided.

To determine the radius of the zone of search we use the Mahalanobis distance

$$d = (\hat{x}(k|k-1) - \hat{x}(k-1|k-1))' P^{-1}(k|k)(\hat{x}(k|k-1) - \hat{x}(k-1|k-1)) \tag{28}$$

that measures the uncertainty of the estimate $\hat{x}(k)$.

All this process can be applied to the Extended Kalman Filter (Restricted Genetic Optimization Filter, RGOF) or a Second-order Nonlinear Filter (Second-order Restricted Genetic Optimization Filter, SRGOF).

5 Comparison of the Nonlinear Filters

A comparison of the nonlinear filters is examined by Monte-Carlo simulations. One set of data $x(k)$ and $z(k)$ are artificially simulated and the estimate $\hat{x}(k)$ is compared with $x(k)$. and the BIAS and the RMSE between the estimated $\hat{x}(k)$ and the simulated one $x(k)$ are computed for each time k. This procedure is performed 6000 times (30 runs of 200 points each).

Fig. 2. Typical executions for the model 5 with EKF and SRGOF for level of noise $\|\varepsilon\| = \|\eta\| = 1.0$

To compare the various estimation methods we consider the five well-known models in ascending order of nonlinearity and with three different levels of noise. In all cases $\varepsilon(k)$ and $\eta(k)$ are assumed to be normally distributed as

$$\begin{pmatrix} \varepsilon(k) \\ \eta(k) \end{pmatrix} \sim \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix} \right), \tag{29}$$

where C is a constant and the initial value $x(0)$ is distributed as a normal random variable.

The first model is linear:

$$\begin{cases} x(k) = x(k-1) + \varepsilon(k) \\ z(k) = x(k) + \eta(k) \quad . \end{cases} \tag{30}$$

The second one is the Logistic Model:

$$\begin{cases} x(k) = \frac{exp(x(k-1)}{exp(x(k-1)) + exp(\varepsilon(k))} \\ z(k) = \frac{exp(x(k))}{exp(x(k)) + exp(\eta(k))} \quad . \end{cases} \tag{31}$$

The third one is the ARCH model:

$$\begin{cases} x(k) = (1 - b + b(x(k-1))^2)^{1/2} + \eta(k) \\ z(k) = x(k) + \varepsilon(k) \quad . \end{cases} \tag{32}$$

The fourth one is the Nonstationary Growth Model given by Kitagawa (1987) and Carlin et al. (1992).

$$\begin{cases} x(k) = 0.5x(k-1) + \frac{25x(k-1)}{1+x^2(k-1)} + 8cos(1.2(t-1)) + \varepsilon(k) \\ z(k) = \frac{x^2(k)}{2}0 + \eta(k) \quad . \end{cases} \tag{33}$$

The last one correspond to the Tracking, with an angle-only sensor given by Bar-Shalom and Fortmann (1988)

$$\begin{cases} x(k) = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} x(k-1) + \varepsilon(k) \\ z(k) = atan\left(\frac{20}{(x(1,k)-x_p(k))}\right) + \eta(k) \quad . \end{cases} \tag{34}$$

Table 1. Comparison of BIAS and RMSE for level of noise $\|\varepsilon\| = \|\eta\| = 0.1$. Each number is the mean of thirty runs of two hundred points each.

	Model 1		Model 2		Model 3		Model 4		Model 5	
	BIAS	RMSE	BIAS	RMSE	BIAS	RMSE	BIAS	RMSE	BIAS	RMSE
EKF	-0.2062	0.9616	0.0058	0.0876	-0.1275	0.9162	0.7118	10.9863	-0.2342	1.1963
IEKF	-0.7654	2.0934	0.0250	0.0822	-1.2593	2.1868	0.4796	10.8942	-0.2488	1.0293
SNF	-0.2063	0.9638	0.0109	0.0536	-0.1210	0.8812	0.4796	10.8942	-0.4361	0.9003
SIF	0.1955	1.9693	8.5E-05	0.0690	-0.1812	1.0947	0.5861	13.1817	-0.3238	0.8980
MSF	-4.9743	36.5653	0.0057	0.0786	-0.1397	0.8718	0.4487	11.3058	-0.2453	0.8828
RGOF	-0.1889	0.9515	0.0076	0.0532	-0.1142	0.8695	0.2688	10.3893	0.2411	1.1523
SRGOF	0.6469	1.3145	0.0078	0.0543	-0.1198	0.8554	0.3893	10.1435	0.0938	0.8511

Table 2. Comparison of BIAS and RMSE for level of noise $\|\varepsilon\| = \|\eta\| = 0.5$. Each number is the mean of thirty runs of two hundred points each.

	Model 1		Model 2		Model 3		Model 4		Model 5	
	BIAS	RMSE	BIAS	RMSE	BIAS	RMSE	BIAS	RMSE	BIAS	RMSE
EKF	-1.9802	4.996	0.2251	1.5537	-0.1691	0.8632	0.2727	14.7955	0.1648	5.5196
IEKF	-6.7135	10.068	0.2233	1.5561	-0.7962	1.7991	0.1858	14.7742	-0.1300	5.0794
SNF	-0.7908	4.354	0.2275	1.5546	-0.1250	0.8463	0.1858	14.7742	-0.0797	4.6296
SIF	0.4687	8.767	0.2269	1.5494	-0.1911	1.0864	0.8650	17.9635	0.1989	4.4094
MSF	-59.0465	154.528	0.2236	1.5540	-0.0628	0.8523	0.5375	14.8041	-0.0202	5.1212
RGOF	-0.8040	3.847	0.2209	1.5525	-0.1453	0.8573	0.4761	10.3697	0.3438	5.9636
SRGOF	-1.1171	4.2704	0.2055	1.5063	-0.1603	0.8611	-1.2549	6.0052	-1.0691	4.8815

Table 3. Comparison of BIAS and RMSE for level of noise $\|\varepsilon\| = \|\eta\| = 1.0$. Each number is the mean of thirty runs of two hundred points each.

	Model 1		Model 2		Model 3		Model 4		Model 5	
	BIAS	RMSE	BIAS	RMSE	BIAS	RMSE	BIAS	RMSE	BIAS	RMSE
EKF	-1.0107	4.8071	0.0811	0.2457	-0.1275	0.9162	0.28454	14.5453	0.2183	11.0601
IEKF	-4.0117	9.7702	0.1628	0.2791	-1.2593	2.1868	0.19430	14.5721	0.2765	9.7927
SNF	-1.0325	4.8194	0.0859	0.2374	-0.1210	0.8812	0.19430	14.5721	0.0084	8.2852
SIF	0.9615	9.8352	0.0369	0.3276	-0.1812	1.0947	0.52118	15.6697	-1.6102	8.2340
MSF	-17.5458	173.3967	0.0810	0.2417	-0.1397	0.8718	0.43729	14.6868	0.6963	8.1326
RGOF	-2.3808	5.5300	0.0524	0.2286	-0.1146	0.8696	0.01695	10.1305	0.3160	10.2766
SRGOF	-2.1398	5.9861	0.0529	0.2287	-0.1198	0.8554	0.11951	9.9886	1.7845	9.1265

6 Discussion

In this paper we have described a new method for filtering nonlinear systems. This method uses Restricted Genetic Optimization to reduce the estimation error of the prior EKF or SNF estimation.

Tables 1-3 summarize the results of the simulations of the five models with three levels of noise: $\|\varepsilon\| = \|\eta\| = 0.1,\ 0.5$ and 1.0. Each number shown in the tables represents the mean of thirty runs of two hundred points each.

It is clear from the above results that the precision (based on the RMS Error criteria) of the filter estimates of the SRGOF are certainly improved over the other algorithms, especially in the noisier and more nonlinear situations. Judging from the criteria of BIAS there are no big differences between the different methods, however some of the algorithms (MSF, SIF and IEKF) are less robust in certain situations.

Finally, the most important charactheristic of the proposed SRGOF algorithm is its robustness in the noisier and highly nonlinear situations which are the desired characteristics of such filters.

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